CONSTRUCTION OF QUADRATURE FORMULAS BASED ON HIGHER-ORDER SPLINES AND THEIR COMPARATIVE ANALYSIS IN SPECIFIC EXAMPLES.

Qobilov S.SH.
Tillaboev A.A.
Muminov E.N.
Teacher of the Department of "Artificial Intelligence" at the Tashkent University of Information Technologies named after Muhammad Al-Khwarizmi
qobilov.sirojiddin92@gmail.com
Teacher of the Department of "Artificial Intelligence" at the Tashkent University of Information Technologies named after Muhammad Al-Khwarizmi
Foraliexpress101@gmail
Teacher of the Department of "Artificial Intelligence" at the Tashkent University of Information Technologies named after Muhammad Al-Khwarizmi
emuminov864@gmail.com
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Abstract:

This study explores the numerical processing of signals using high-order Ryabenky spline models within the framework of approximation theory. Third-, fifth-, and seventh-order spline models were selected, with a focus on their efficiency in digital signal processing. Experimental gastroenterological signal data were used to build approximations and perform comparative analyses. Results show that the seventh-order Ryabenky spline provides the most accurate approximation among the tested models. These high-order spline models can also be applied to other types of medical and digital signals.

Keywords: digital signal processing, spline models, mathematical modeling, medical signals, gastroenterological signals.

Introduction:

Modern medical research increasingly emphasizes accurate monitoring and early disease detection, with digital signal processing playing a key role. The integration of fields such as mathematics, medicine, and technology highlight the value of spline-based approximation methods. Spline functions have proven to be powerful tools for modeling and signal reconstruction, especially in complex domains like gastroenterology. Motivated by this, the present work applies Ryabenky spline functions to process real-world medical signals and evaluate their effectiveness. If the function under consideration is sufficiently smooth, then it is desirable to approximate it with splines of a higher degree so that the convergence is faster. For this purpose, the error of approximating the function by splines of the seventh degree of Ryaben'kiy has been investigated. Continuing

the study, the splines of the seventh degree of Ryabensky were obtained [1,4,5,6].

In this paper, using quadrature formulas, we will calculate estimates of their error.

Let p>0 be a fixed integer, $f(x) \in C^q(-\infty,\infty)$ and $f(x) = f(ih), (i = 0,\pm1,\pm2,...)$ the trace of a function on a uniform one-dimensional grid [4,5,9].

Ryaben'kii's papers describe the operator L that maps $f(ih)$ to the function Sph(x). defined everywhere and being an interpolation spline of degree 2p+1 of defect p+1. To estimate the error in the one-dimensional case with respect to the error $f(x)$ - Sph(x) and order derivatives, the following estimate follows:

$$\left|f^{(r)}(x) - S_{ph}^{(r)}(x)\right| \le h^{q-r} K(p) \sup_x \left|f^{(q)}(x)\right|. \qquad (1)$$

where the constant $k(p)$ depends only on p.

For any spline $S_{ph}(x)$ can be written explicitly in terms of the values of the function $f(ih)$, so it is convenient for applications. In contrast to Hermitian splines, these splines can also be used to approximate C[a,b] functions.

In the works of Sodolev and Ryaben'kiy, for $x_i, \le x \le x_{i+1}$ the following expression is given for the function $S_{ph}(x)$ [3,6]

$$S_{ph}(x) = \sum_{j=0}^{p} \sum_{k=0}^{p-j} \frac{(p+k)!}{k!\,j!\,p!} \left[a_{pj}(x_i) t^{j+k} (1-t)^{p+1} + (-1)^j a_{pj}(x_{i+1}) t^{p+1} (1-t)^{j+k}\right] \qquad (2)$$

где $t = \dfrac{x - x_i}{h}$, $a_{p0}(x_i) = f(x_i)$

$$a_{pj} = j! \sum_{k=j}^{p} \frac{\Delta f(x_i)}{k!} S(k, j) \quad j \ge 1 \qquad (3)$$

The spline of the third degree of Ryabenky [2,4,5,9], on the segment $[x_i; x_{i+1}]$

$$S_3(x) = \varphi_1(t) f(x_i) + \varphi_2(t) f(x_{i+1}) + \varphi_3(t) f(x_{i+2}). \qquad (5)$$

here: $\varphi_1(t) = (1-t)^2(1+t), \quad \varphi_2(t) = t(1+2t-2t^2), \quad \varphi_3(t) = -t^2(1-t)$

p=1

$$S_{1h}(x) = \sum_{j=0}^{1} \sum_{k=0}^{1-j} \frac{(1+k)!}{k!\,j!\,1!} \left(a_{1_j}(x_i) t^{j+k} (1-t)^{1+1} + (-1)^j a_{1_j}(x_{i+1}) t^{1+1} (1-t)^{j+k}\right) =$$

$$= a_{10}(x_i)(1-t)^2 + a_{10}(x_{i+1}) t^2 + 2a_{10}(x_i) t (1-t)^2 + 2a_{10}(x_{i+1}) t^2 (1-t) +$$

$$+ a_{11}(x_i) t (1-t)^2 - a_{11}(x_{i+1}) t^2 (1-t) =$$

$$= a_{10}(x_i)(1-t)^2 (1+2t) + a_{10}(x_{i+1}) t^2 (3-2t) + a_{11}(x_i) t (1-t)^2 -$$

$$- a_{11}(x_{i+1}) t^2 (1-t)$$

$$t = S_{1h}(x) = a_{10}(x_i)(1-t)^2(1+2t) + a_{10}(x_{i+1})t^2(3-2t) +$$
$$= a_{11}(x_i)t(1-t)^2 - a_{11}(x_{i+1})t^2(1-t)$$
$$a_{10}(x_i) = f(x_i),\ \Delta f(x_i) = f(x_{i+1}) - f(x_i)$$
$$a_{11}(x_i) = 1! \sum_{k=1}^{1} \frac{\Delta f(x_i)}{1!} S(1,1) = f(x_{i+1}) - f(x_i),\ S(1,1) = 1$$
$$S_{1h}(x) - S_3(x) = f(x_i)(1-t)^2(1+2t) + f(x_{i+1})t^2(3-2t) +$$
$$+\left(f(x_{i+1}) - f(x_i)\right) + (1-t)^2 - \left(f(x_{i+2}) - f(x_{i+1})\right)t^2(1-t) =$$
$$= \left((1-t)^2(1+2t)t(1-t)^2\right) f(x_i) + \left(t^2(3-2t) + t(1-t)^2 + t^2(1-t)\right) f(x_{i+1}) -$$
$$-t^2(1-t) f(x_{i+2}) = \phi_1(t) f(x_i) + \phi_2(t) f(x_{i+1}) + \phi_3(t) f(x_{i+2})$$
$$\phi_1(t) = (1-t)^2(1+t),\ \phi_2(t) = t(1+2t-2t^2),\ \phi_3(t) = -t^2(1-t)$$

Above, we considered the construction of the third-order Ryabenikov spline function[4,5,9], and thus the 5th and 7th-order models of this spline function were also constructed. The constructed models are shown in Figure 1 in the Maple program. Now we will consider the application and comparative analysis of 3rd, 5th and 7th order Ryabenikov spline functions to gastroenterological signals.

$$SR_3(x) = h\sum_{i=1}^{N} f(ih) + \frac{5}{12}h\left(f(0) - f(1)\right) + \frac{1}{12}h\left(f(h) - f(1+h)\right) \quad (6)$$

here: $f(x),\ x = ih,\ i = 1...N$

$$SR_5(x) = h\sum_{i=1}^{N} f(ih) + \frac{43}{120}h\left(f(0) - f(1)\right) + \frac{1}{5}h\left(f(h) - f(1+h)\right) - \frac{7}{120}h\left(f(2h) - f(1+2h)\right)$$

here: $f(x),\ x = ih,\ i = 1...N$

(7)

$$SR_7(x) = h\sum_{i=1}^{N} f(ih) + \frac{549}{1680}h\left(f(0) - f(1)\right) + \frac{523}{1680}h\left(f(h) - f(1+h)\right) -$$
$$-\frac{313}{1680}h\left(f(2h) - f(1+2h)\right) + \frac{81}{1680}h\left(f(3h) - f(3+h)\right) \quad (8)$$

here: $f(x),\ x = ih,\ i = 1...N$

Table 1 presents the comparative analysis results of the $f(x) = x^7$ function selected as an experiment on the basis of the spline functions of the 3rd, 5th, and 7th orders given above with integral values at different values of N.

Table 1

	f (x)	SR3(x)	SR5(x)	SR7(x)	f(x)-SR3(x)	f(x)-SR5(x)	f(x)-SR7(x)
10	0,125	0,12289827	0,12310161	0,1239654	2,10E-03	1,90E-03	1,03E-03
20	0,125	0,12476026	0,124847401	0,124955721	2,40E-04	1,53E-04	4,43E-05
30	0,125	0,1249311	0,124961806	0,12499238	6,89E-05	3,82E-05	7,62E-06
40	0,125	0,12497137	0,124985271	0,124997755	2,86E-05	1,47E-05	2,24E-06
50	0,125	0,12498548	0,124992869	0,12499912	1,45E-05	7,13E-06	8,801E-07
60	0,125	0,12499165	0,124996028	0,124999588	8,35E-06	3,97E-06	4,121E-07
70	0,125	0,12499476	0,124997567	0,124999782	5,24E-06	2,43E-06	2,177E-07
80	0,125	0,1249965	0,124998404	0,124999874	3,50E-06	1,60E-06	1,256E-07
90	0,125	0,12499755	0,124998898	0,124999226	2,45E-06	1,10E-06	7,742E-07
00	0,125	0,12499822	0,124999207	0,124999497	1,78E-06	7,9302E-07	5,029E-07

As can be seen from the table above, the results obtained using higher-order spline functions are very close to the results of the function obtained as an experiment. This means that the application of the selected model in the field of medicine will give us good results. In the next section, we consider the application of the model to gastroenterological and codeological signals and their comparative analysis. It is known to us that in the field of cordiology, it is very important to accurately calculate the value of the heartbeat amplitude in monitoring human health and diagnosing heart diseases. Gastroenterological signals are also of great importance in determining the diseases of the human internal organs and in evaluating the effect and process of the treatment procedures applied to them. Table 2 presents the values of gastroenterological signals and their comparative analysis restored on the basis of the higher-order spline function.

Table 2.

	f(x)	*SR3(x)*	*SR5(x)*	*SR7(x)*	*f(x)-SR3(x)*	*f(x)-SR5(x)*	*f(x)-SR7(x)*
10	0,25	0,24975	0,2499	0,25	2,50E-04	1,00E-04	0
20	0,25	0,24996875	0,2499875	0,25	3,13E-05	1,25E-05	0
30	0,25	0,24999074	0,249996296	0,25	9,26E-06	3,70E-06	1E-15
40	0,25	0,24999609	0,249998438	0,25	3,91E-06	1,56E-06	1E-15
50	0,25	0,249998	0,2499992	0,25	2,00E-06	0,0000008	1E-15
60	0,25	0,24999884	0,249999537	0,25	1,16E-06	4,62963E-07	0
70	0,25	0,24999927	0,249999708	0,25	7,28863E-07	2,91545E-07	0
80	0,25	0,24999951	0,249999805	0,25	4,88281E-07	1,95313E-07	0
90	0,25	0,24999966	0,249999863	0,25	3,42936E-07	1,37174E-07	1E-15
100	0,25	0,24999975	0,2499999	0,25	0,00000025	0,0000001	0

From the results obtained from Table 2, it can be seen that from the higher-order spline functions, the seventh-order spline function returned the values for

which the values of the function were the least different from the true values. So digital processing of signals through this model gives good results in the considered medical field. You can see this from the results in the table above.

Conclusion:

In this article, we considered the construction of spline functions of the third, fifth, and seventh degrees. In addition, a comparative analysis was conducted with the function values obtained as an experiment. Based on the obtained results, the effectiveness of the considered model was evaluated, its application to gastroenterological signals and a comparative analysis was presented. From the obtained results, the maximum value of the differences of the results obtained by the spline function of the third degree from the actual values was equal to 2.5E-04, and in the spline function of the fifth degree, it was equal to 1.0E-04. Among the considered models, the spline with the highest efficiency and accuracy is the seventh-order spline function, and it is not difficult to see this from the obtained results. This is of great importance in digital processing of medical signals.

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