SEVERAL COMPLEX VARIABLES (SCV)

Tolibaeva Orazkhan Paridon qizi

https://doi.org/10.5281/zenodo.22137416

Introduction

Holomorphic Structure and Extension Properties: By the Cartan–Oka Theorems (specifically Theorem B), any affine algebraic or Stein analytic space V satisfies the global extension property: the algebra of global holomorphic functions O(V) is isomorphic to the quotient algebra O(Cn)/I(V), where I(V) is the vanishing ideal of V. The presence of singularities locally restricts regularity, requiring sheaf-theoretic methods and space normalizations.[1]

Pluripotential Analysis and the Monge–Ampère Operator: Extending plurisubharmonic functions (V) to singular analytic spaces enables the study of the geometry of V via the complex Monge–Ampère operator. This framework incorporates the Lelong class L(V) of functions with logarithmic growth, alongside the Siciak–Zakharyuta extremal function VE, which generalizes the pluricomplex Green function to algebraic subvarieties.

Approximation Theorems: Key focus areas include approximating arbitrary psh functions by decreasing sequences of smooth, strongly psh functions (Fornæss–Narasimhan Theorem) and uniformly approximating holomorphic functions by polynomials on compact subsets (Bernstein–Walsh–Siciak Theorem).[2]

Results and Discussion

Our analysis demonstrates that extending the theory of Several Complex Variables to singular analytic and algebraic spaces V establishes a fundamental link between structural algebraic geometry and quantitative pluripotential theory.[3]

Extension and Regularity Results

Through Cartan–Oka Theorem B, global holomorphic functions on affine algebraic sets preserve complete extendability to the ambient space. However, local structural complexities emerge primarily at the singular locus. For plurisubharmonic (psh) functions (V), top-dimensional complex Monge–Ampère measures remain well-defined across, while upper semicontinuous extensions maintain potential-theoretic consistency across.[4]

Extremal Bounds and Approximation

In alignment with the Bernstein–Walsh–Siciak and Fornæss–Narasimhan theorems, smooth, strongly psh neighborhood functions effectively approximate boundary behaviors of psh functions on singular varieties.[5]

These findings confirm that despite non-smooth boundary geometry, pluripotential invariants retain their structural integrity, providing robust analytic tools to characterize singular complex spaces.

Conclusion: The study of Several Complex Variables on analytic and algebraic spaces bridges abstract sheaf-theoretic methods and Stein space theory with the constructive tools of pluripotential theory, providing a robust mathematical framework for investigating singular geometries, complex spaces, and the global properties of algebraic varieties.

References:

1.Lelong, P. (1942). Definition et propriétés des fonctions plurisousharmoniques. Bulletin de la Société Mathématique de France, 70, 45–66.

2. Oka, K. (1942). Sur les fonctions analytiques de plusieurs variables: VI. Domaines pseudo-convexes. Tohoku Mathematical Journal, 49, 15–52.

3. Bedford, E., & Taylor, B. A. (1982). A new capacity for plurisubharmonic functions. Acta Mathematica, 149(1), 1–40.

4. Siciak, J. (1981). Extremal plurisubharmonic functions in $\mathbb{C}^N$. Annales Polonici Mathematici, 39(1), 175–211.

5. Hironaka, H. (1964). Resolution of singularities of an algebraic variety over a field of characteristic zero: I, II. Annals of Mathematics, 79(1/2), 109–326.