



GEOMETRIC AND PHYSICAL MEANING OF THE DERIVATIVE

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Abstract. This article provides a detailed explanation of the geometric and physical meaning of the concept of derivative. From a geometric point of view, the derivative is interpreted as a gradient that determines the direction of the tangent line at each point of the curve. In physics, the derivative is often used to express quantities such as motion, velocity, and acceleration. The article reveals the importance of the derivative through real-life examples along with theoretical concepts. It also discusses the practical application of the derivative and what methods can be effective in explaining it to students in education.

Keywords: Derivative, tangent, gradient, velocity, acceleration, physical meaning, geometric interpretation, differential, curve, analysis.

Today, the educational process is required to be closely related to the development of science and technology. However, this does not mean that students should be taught all the innovations created by mankind. Each educational institution should select the materials it needs from the achievements of science and technology. One of such important topics is information about the integral and integral of a function. The geometric and physical content of these concepts is covered in many literatures, and they are effectively used in the educational process.

As a result of the development of mathematics, the concepts of the integral and derivative of a function have also appeared, and scientific research is being conducted on their properties. Today, it is becoming urgent to introduce these concepts into the educational system. In particular, it is advisable to teach these topics through special courses or additional lessons in academic lyceums and higher educational institutions. This can develop students' logical thinking skills and increase their interest in science.

During its development, humanity, while providing knowledge to the younger generation, tries to teach the basics of sciences, taking into account the requirements of its own activities and development. Therefore, the task of providing students with in-depth knowledge of mathematics, along with all other knowledge, and its scientific implementation are among the main issues.



In this regard, the methodology of teaching mathematics occupies one of the main places. The word "methodology" is derived from the Greek word "method" or "method". The science of mathematics teaching methodology (method) is called the branch of pedagogy that studies and researches the methods and laws of teaching mathematics at a certain level of its development in accordance with the educational goals set by society. Mathematical methodology is one of the main branches of pedagogy and didactics, and is an independent science that studies the laws of teaching and learning mathematics that correspond to the goals of education at the level of development of our society.

The concept of mathematical methodology was first described in the work of the Swedish pedagogue and mathematician G. Pestalozzi, written in 1803, "The Demonstrative Study of Numbers." Starting from the first half of the 17th century, issues related to the methodology of teaching mathematics were addressed by Russian scientists, such as Academician S.E. Guriev (1760-1813), and from the first and second half of the 18th century, by N.I. Lobachev (1792-1856), I.N. Ulyanov (1831-1886), L.N. Tolstoy (1828-1910), and the outstanding methodologist-mathematician S.I. Shokhor-Trotsky (1853-1923), A.N. Ostrogradsky, and others. They looked at mathematics from a scientific point of view and developed its progressive foundations. For example, A.N. Ostrogradsky wrote: "Consciousness arises after observation, consciousness "based on the real, existing world."

In addition, in covering the topic, students are shown practical application aspects by explaining its geometric and physical content. This helps students understand the importance of science in life. The derivative is one of the basic concepts in mathematics, which is widely used in mathematics, physics and other sciences to analyze various problems, in particular, the speed of various processes.

In mathematical analysis, the derivative is one of the basic concepts that determines the rate of change of a variable quantity. Understanding the theoretical foundations of the derivative allows us to apply it to various practical problems. In this article, we will consider the essence of the derivative from two main perspectives - geometric and physical.

The derivative is the rate of change of a function at a certain point. Geometrically, it is the slope of the tangent line drawn to the curve corresponding to the function at this point. By determining how the derivative changes, you can analyze how the graph will look.



Differential calculus is a branch of mathematics that deals with calculating derivatives and differentials, studying their properties, and applying them to verifying functions.

The initial work on the emergence of differential calculus was done by Fermat, Descartes, and other mathematicians to solve the problem of passing a curve. I. Newton and G. Leibniz completed the work of their predecessors in this area.

In physics, the derivative is often expressed in terms of time-dependent changes (distance - velocity - acceleration). For example, by expressing the change in time of moving objects through the derivative, it is possible to determine their speed at a given time.

Practical examples include:

- Determining how a car's speed changes over time.
- The rise or fall of a water level.
- Analyzing shifts in supply and demand curves in economics.

Also, Velocity and Acceleration: In physics, the derivative is related to many important quantities. For example, the velocity of a moving object is a measure of the change in its position with respect to time. Acceleration is the change in velocity with respect to time. These formulas help describe the dynamics of motion of objects.

Example of Physical Kinematics: By observing how the path and velocity of a car change over time, it is possible to determine whether its acceleration is constant or variable.

Finding Optimal Points: The derivative can be used to find the maximum or minimum values of a function. For example, in business, it is used to determine the points of maximum revenue or minimum cost. To find the maximum or minimum points of a function, the derivative must be zero.

Critical Points: The derivative of a function is zero and other conditions can be used to determine the location of a function that is constant or has changed. These critical points are useful for situations that are analyzed in business and economics. Learning about velocity and acceleration: When talking about the motion of an object, if the position of the object depends on time, we can find its change with time to find its velocity.

Knowing the velocity, we can determine acceleration. We can calculate the speed of a car with respect to time to see how its speed changes.



Geometric Analysis of Paths: From a geometric perspective, the derivative is useful in fields such as education, construction, and architecture, by determining the slope of lines and curves.

Using visual graphs and real-life examples (such as car motion) to teach students the derivative is very effective. Through practice, exercises in drawing tangents to curves make it easier to understand the topic.

In the second problem that leads to the concept of the derivative, we saw that the instantaneous velocity of a material point moving along a straight line, whose law of motion is described by the function $s=s(t)$, at time t is $v\omega$. From this follows the physical (mechanical) meaning of the derivative. In rectilinear motion, described by the function $s=s(t)$, the numerical value of the velocity at time t is equal to the derivative: $v\omega = s'(t)$. The mechanical meaning of the derivative can also be briefly stated as follows: the derivative of the path with respect to time is equal to the velocity. The concept of the derivative allows us to determine not only the instantaneous velocity of rectilinear motion, but also the instantaneous velocity of other processes. For example, suppose $y=Q(T)$ is a function describing the change in the amount of heat transferred to heat a body to a temperature T . In this case, the heat capacity of the body is equal to the product of the amount of heat with respect to temperature: C . In general, the derivative can be said to be a mathematical model of the instantaneous rate of a process described by the function $f(x)$.¹

The process of finding the derivative of a function is called differentiation of a function. If a function is defined on an interval and has a derivative at each point in this interval, then the derivative formula defines the derivative as a function on this interval. If a specific point is not specified in the differentiation, we find the derivative at all available points of the function, and in this case we denote it as .

In the case of the slope of a curve, the slope of the slope was determined by the following equation. This equation expresses the geometric meaning of the derivative, that is, the derivative is equal to the slope of the slope of the curve at a given point on the graph of the function.

The process of finding the derivative of a function is called differentiation. If a function is defined on a certain interval and has a derivative at each point in this interval, the derivative is considered a new function and is defined as.

¹ Rasulov M.A. – Matematik analiz, Toshkent: Fan va Texnologiya, 2012.



Also, in chemistry, the derivative of a function indicates the rate of change of the reacting substance with respect to time. If there is a function expressing the rate of reaction, then it expresses the rate of reaction at a given moment in time.

Through the derivative of a function, it is possible to analyze many physical, geometric, and chemical processes in real life using mathematical models and determine changes in them. The derivative represents the rate of change of these processes, and the graph represents the slope of curves determined by geometry.

Also, in the study of rectilinear motion, the following result was reached through the limit: the derivative taken with respect to time represents the speed of rectilinear motion of this material point. This situation indicates the mechanical meaning of the derivative.

In general, if a function expresses some physical process, then the derivative represents the rate of occurrence of this process. This idea explains the physical meaning of the derivative.

For example, consider the rate of a chemical reaction: in this case, the function expresses the amount of a reactant as a function of time. As time increases, the change in the amount of this substance also increases. The ratio of these changes gives the average rate of the chemical reaction. If this ratio approaches zero through the limit, that is, if the time intervals are infinitely small, the resulting limit function expresses the rate of reaction at a point.

Suppose that $y = f(x)$ is a complex function, that is, a function is given that can be described in the form $y = F u$, $u = \varphi x$, or $y = F \varphi(x)$. In the expression $y = F u$, this variable is called the intermediate argument. Theorem. If the function $u = \varphi x$ has a derivative $u x' = \varphi'(x)$ at some point x , and the function $y = F u$ has a derivative $y x' = F'(u)$ at the corresponding value of u , then at the indicated point x the complex function $y = F \varphi(x)$ also has a derivative equal to $y x' = F u'(u) \varphi'(x)$, where u should be replaced by the expression $u = \varphi x$. In short, $y x' = y u' u x'$, that is, the derivative of a complex function is equal to the product of the derivative of the given function with respect to the argument u in the interval by the product of the argument with respect to x in the interval.

Also, Curve and Tangent: In geometric terms, the derivative is the slope of the tangent line to the curve of a function. The derivative at each point is interpreted as the slope of the tangent line to the curve. The tangent line is the line that is closest to the curve at a point.



Difference between tangent lines: If we calculate the derivative at several points, the slope of the tangent line at each point does not change. The derivative shows the change in the function and allows us to determine what the graph will look like.

It is effective to give real-world examples using graphs to explain the derivative to students. For example, graphs showing changes in the speed of a car or changes in the weather make it easier for students to understand.

Experiments and experiments: Experiments can be conducted to show students the physical meaning of phenomena. For example, the movement of an object can be observed and analyzed through velocity and acceleration.

In conclusion, the derivative is an important concept not only in mathematics, but also in physics and many other fields of science. Geometric and physical interpretations help to understand it more deeply. This serves as the basis for teaching it and applying it to practical problems.

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