COMPLEX ANALYTIC AND PLURISUBHARMONIC PROPERTIES OF ALGEBRAIC VARIETIES

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Introduction (English)

The study of complex analytic and plurisubharmonic properties on algebraic varieties lies at the intersection of complex geometry, potential theory, and commutative algebra. Algebraic varieties provide a natural geometric framework where function-theoretic behavior is tightly constrained by underlying polynomial equations. Over the past few decades, understanding how holomorphic functions and plurisubharmonic (psh) potentials extend, degenerate, or aggregate singularities along subvarieties has become central to multidimensional complex analysis.[1]

While classical potential theory focuses heavily on Euclidean domains, extending these concepts to singular or non-compact algebraic sets introduces significant analytical challenges. Key invariants such as Lelong numbers, Monge–Ampère measures, and extremal psh functions reveal subtle connections between the local topology of a variety and its global analytic capacity.

This paper examines the structural interaction between algebraic geometry and pluripotential theory. By analyzing the growth rates and boundary values of plurisubharmonic functions, we aim to establish novel extension theorems and polynomial approximation bounds on algebraic varieties.[2]

Methodology (English)

This research employs a synthesis of pluripotential theory, algebraic geometry, and geometric function theory. First, we utilize the generalized complex Monge–Ampère operator $(dd^c u)^n$ on smooth and weakly singular algebraic varieties to measure the density of plurisubharmonic (psh) singularities. Second, we integrate Lelong numbers and greenian potentials to evaluate local boundary growth conditions and metric capacity. Third, polynomial approximation techniques are applied alongside classical Cauchy–Riemann ($\bar{\partial}$) L^2-estimates to determine continuous extension criteria for holomorphic functions. Finally, global properties are established by mapping localized psh functions onto projective embeddings using Bedford–Taylor capacity measures and logarithmic potential bounds.[3]

Results (English)

This research yields four main analytical results regarding functions on complex algebraic varieties:

1. **Singular Extension Theorem:** Plurisubharmonic (psh) functions bounded near singular loci Sing(V) extend uniquely to V, with their Monge–Ampère mass vanishing over (V), proving that algebraic singularities are pluripolar sets.

2. **Lelong Mass Bounds:** The Lelong number of a logarithmic polynomial potential at a singular point V is scaled by the geometric multiplicity of the variety v = m, while total Monge–Ampère mass remains bounded by (V).

3. **Algebraic Identity Principle:** Bernstein quasi-analytic functions defined on compact non-pluripolar subsets of V satisfy an identity principle, propagating vanishing properties across the variety.

4. **Weighted Holomorphic Extension:** Using estimates for the operator, we established continuous extension bounds for holomorphic functions from V with growth controlled by plurisubharmonic weights.[5]

Conclusion (english)

This study establishes fundamental analytical properties of holomorphic and plurisubharmonic functions on complex algebraic varieties. By extending the Bedford–Taylor Monge–Ampère operator to singular subvarieties, we demonstrated that singular loci form pluripolar sets that do not charge complex Monge–Ampère mass. Furthermore, we quantified the density of singularities using Lelong numbers and derived optimal upper bounds for logarithmic growth potentials based on the variety's algebraic degree. The general identity principle for Bernstein quasi-analytic functions and the weighted L^2-extension theorems confirm that algebraic geometry exerts strong structural control over complex potential theory, offering crucial tools for multidimensional complex dynamics.

References:

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