THE EMPIRICAL CHARACTERISTIC FUNCTION AND LARGE SAMPLE HYPOTHESIS TESTING

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Annotation: Many properties of distribution functions can be characterized in terms of their characteristic functions. This suggests that statistical inference about such properties might utilize the characteristic function. This requires that some means be found to gather information about the characteristic function from a random sample. In this investigation, the empirical characteristic function is proposed to do this.

Key words: empirical characteristic function, characteristic function, probability space, independent and identically distributed random variables.

INTRODUCTION.

Let (Ω, ς, P) be a fixed but otherwise arbitrary probability space. Let $X_1, X_2, \ldots. X_n$ be independent and identically distributed (i.i.d) random variables defined on Ω with common distribution function F and characteristic function c. Then the empirical distribution function is $F_n(x) = N(x)/n, \quad x \in R,$ where N(x) is the number of $X_j \le x$ for $1 \le j \le n$. The empirical characteristic function (e.c.f) is defined by

$$c_n(t) = \int e^{itx} dF_n(x) = \frac{1}{n} \sum_{j=1}^{n} e^{itX_j}, \ t \in R$$

To justify the use of the e.c.f. for statistical inference we should know something about the convergence of the e.c.f. to the characteristic function. In this chapter we shall discuss this convergence briefly and give a class of hypothesis testing problems to which the e.c.f. may be applied.

CONVERGENCE OF THE E.C.F.

We first note that for fixed t, $c_n(t)$ is average of bounded i.i.d. random variables. Therefore, by the strong law of large numbers $c_n(t)$ converges almost surely to $c(t)$ for every $t \in R$. Furthermore, we have the following result of Feuerverger and Mureika (1977).

THEOREM. For fixed $T \prec \infty$,

$$P\left\{\lim_{n\to\infty}\sup_{|t|\le T}\left|c_n(t)-c(t)\right|=0\right\}=1$$

Let $X_1, X_2, ...X_n$ be independent and identically distributed random variables with common law X. For given μ *and* σ^2 consider the problem of testing $H_0 : X \sim \eta(\mu,\sigma^2)$. This hypothesis is equivalent to $H_0 : X' \sim \eta(0,1)$ *where* $X' = (X-\mu)/\sigma$. Hence without loss of generality we will take $\mu = 0$ *and* $\sigma^2 = 1$.

Let $c_n(t)$ be the e.c.f. of $X_1, X_2, ...X_n$. Noting that $\exp\left(-\frac{1}{2}t^2\right)$ is the characteristic function of a $\eta(0,1)$ distribution, we take as our test statistic the following

$$s_n = \sup_{t\in S}\left|c_n(t)-\exp\left(-\frac{1}{2}t^2\right)\right|$$

where S is a bounded subset of the real line. Here we will take S to be the points of maximization of

$$\lambda(t)=\left|\exp\left(-\frac{1}{2}t^2\right)-c*(t)\right|$$

Where $c*(t)$ is the characteristic function of some close alternative distribution. Taking $c*(t)$ to be the characteristic function of a $\eta(\mu,1)$ distribution with μ chosen small, we then have

$$\lambda(t)=\left|\exp\left(-\frac{1}{2}t^2\right)-\exp\left(i\mu t-\frac{1}{2}t^2\right)=\exp\left(-\frac{1}{2}t^2\right)\right|\left|-\exp(i\mu t)\right|$$

$$=\exp\left(-\frac{1}{2}t^2\right)\left\{2-2\cos(\mu t)\right\}^{\frac{1}{2}}$$

It can be seen that the e.c.f. based test performs significantly better than the Kolmogorov-Smirnov test for these alternatives. This is to be expected since the point t_0 was chosen to make the test sensitive to location shift alternatives. However it is significant that the e.c.f. test performs better than the Kolmogorov-Smirnov test for values of the mean less than 0.1 since that is the specific alternative used in determining t_0.

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